

An Ancient Method of Finding and Extending Direction

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The ability of the ancient Egyptian surveyors to orient some of their monuments to the meridian with accuracy has long perplexed modern societies. Although generally their temples were not well placed, the pyramids, and especially the Great Pyramid, were oriented almost precisely to True Geographical North.

To account for this, various methods have been proposed which range from pure chance to a precise star measuring system. I believe, however, the means used for orienting the pyramids was actually based on the movements of the sun. Many ancient cultures have made use of the sun's movements by measuring its shadows with an instrument called a gnomon, a pole placed vertically on the ground. Ancient pictographs show the Egyptians, too, made use of this instrument. With the gnomon and a notched device called a bay (once considered to be a means for sighting distant stars), they were able to read the shadows with precision. In addition, the direction found by this gnomon, although short, can be extended accurately by use of the "Stretching of the Cord" ceremony, a foundation-laying ritual.

The state of ancient Egyptian technology indicates that the pyramid builders had access to only the most rudimentary devices. For example, the magnetic compass, which we take for granted, was unknown to them. Indeed, if they had been acquainted with its use, the magnitude of error would have been much greater, for the north magnetic pole, which the needle seeks, is not on the meridian as is the geographical pole but varies according to the place and the year the reading is taken. In addition, their knowledge of astronomy was at a very elementary

level, their greatest astronomical achievement being the calendar.¹ Whatever means were used by the ancient surveyors, the results are truly remarkable.

The actual orientation of the four sides of the Great Pyramid is:²

TABLE 1 Orientation of the Sides
of the Great Pyramid

West side	2'30" W. of N.
East side	5'30" W. of N.
North side	2'28" S. of W.
South side	1'57" S. of W.

It has been suggested that this accurate orientation could only have been achieved by using the celestial bodies;³ and several possible solutions have been offered. Engelbach feels it might have been accomplished by sighting on a star and bisecting the angle between the rising position, the observer, and the setting position. Alternatively, he feels it is possible the Egyptians took their observations on the extreme positions of a circumpolar star.⁴

Edwards also suggests this could only have been achieved with the aid of one or more of the celestial bodies, and goes on to say:

¹ R. A. Parker, "Egyptian Astronomy, Astrology, and Calendrical Reckoning," *Dictionary of Scientific Biography*, Supplement 1 (New York, 1978), 727.

² S. Clark and R. Engelbach, *Ancient Egyptian Masonry* (London, 1930), 68.

³ Z. Žaba, *L'Orientation Astronomique dans L'Ancienne Égypte, et la Précession de L'Axe du Monde* (Prague, 1953), 13.

⁴ Clark and Engelbach, op. cit., 68.

It was clearly only necessary to fix one axis; the other axis could be determined by the use of a set square; contemporary buildings with corners forming a perfect right angle prove that an accurate instrument for this purpose must have been available to the pyramid builders. East and west could have been discovered approximately by observing the rising and setting of the sun on the two equinoctial days every year, and north by an observation of the Pole Star, but in each case the resultant error (even after allowance has been made for a change in the position of the pole in relation to the pole star in the course of about 4,500 years) would have been greater than the amount revealed by at least the two main pyramids at Giza. To judge from the instrumental and representational evidence so far found, it seems more likely that the high degree of accuracy was achieved by astral than by solar observation.⁵

Edwards proposes a method to find true north by building a circular wall higher than a man whose top is an absolute plane. This wall would form an artificial horizon to an observer standing in its center. This observer, while sighting over a pole or through the slit of a bay, would direct another to mark the position of a star as it rose above the wall, and again, when the star set. True north may be obtained, he claims, with a line taken between the bisection of these two points and the center pole.⁶

Having performed a number of tests which utilized a similar technique, and will be subsequently described, I am led to question the feasibility of the Edwards theory. The difficulty of finding accurate direction by marking and bisecting a simple circle is troubling enough, without the additional burden of building a perfectly circular wall, higher than a man, whose top is an absolute plane.

In an exhaustive study, Žaba states that there are two possible astronomic methods of finding true north: using the shortest shadows cast by

the sun to establish the meridian⁷ or bisecting extreme east-west positions of a circumpolar star⁸ (there was no polar star in ancient Egypt at the time, the closest being Alpha Draconis (Thuban) which was 1° 40' from the pole).⁹ Claiming that the solar method has inherent difficulties that would cause it to be inaccurate, Žaba selects Eta of the Great Bear, as the target star. To bisect its circumpolar path, he suggests a combination of a stable plumb line (*mrht*) and a sighting instrument (*psš-kf*) mounted on a block of wood. This device is guided by a slat on a table, so that it can be shifted to the star's east-west extremities, which are then bisected. The east-west axis of the slat may be found on the previous day, he suggests, by using the same method or by a study of the shortest shadow of the sun. However, he states, finding the axis in either manner would give only an approximation of the east-west direction.¹⁰ It is unusual to seek accuracy with a procedure that is inexact. In addition, assuming some such combination of sliding or swinging sight lines can be made to produce satisfactory results it seems very sophisticated compared to other ancient Egyptian technology. Indeed, Žaba himself states there is no evidence suggesting the use of this method.¹¹

In laying out the structure, he feels, as does Edwards, it is only necessary to establish one of two directions (north-south or east-west) and by transferring the lines and using basic geometry, it may be carried over to all of its sides. However, he feels, the mechanics of this operation, according to the care given and the size of the base, must result in an escalation of error even though the desired astronomic orientation may have been determined with strict accuracy. Therefore, he claims, the known deviations are probably not a result of imperfect astronomic observation, but rather, faulty geometric operations.¹² This presumption of a very precise as-

⁵ I. E. S. Edwards, *The Pyramids of Egypt* (New York, 1985), 265.

⁶ *Ibid.*, 267, fig. 56.

⁷ Žaba, *op. cit.*, 64.

⁸ Žaba, *op. cit.*, 70-71.

⁹ E. C. Krupp, "Great Pyramid Astronomy," *Griffith Observer*, Vol. 42, No. 3, p. 9 (Griffith Observatory, Los Angeles, California, 1978).

¹⁰ Žaba, *op. cit.*, 71.

¹¹ Žaba, *op. cit.*, 71.

¹² Žaba, *op. cit.*, 14.

tronomic method and inexact geometric method may seem quite logical; however, it may not be true. The possibility exists that the geometric operations can be done with accuracy; but finding the direction, because of natural phenomena upon which it is based, causes errors to occur.

It is logical that only one direction need be found, for the other can be found geometrically; and there is general agreement on this point. The accuracy displayed in the Egyptians' geometric operations is evident in the corners of the Great Pyramid:¹³

TABLE 2 Corner Angles of the Great Pyramid

Corner			Deviation From Right Angle
North-east	90° 3'20"	+ 3'20"
South-east	89° 56'67"	- 3'33"
South-west	90° 0'33"	+ 0'33"
North-west	89° 59'58"	- 0'20"

It is worth noting that the average deviation produced by the orientation process in the Great Pyramid as shown in Table 1 is 0°4'0" west of true north, while Table 2 shows the average deviation produced by all the corners, using the geometric method, is 0°1'47". In this instance at least, the superiority of the geometric method is clear.

The primary purpose of the Egyptian study of the heavens was the reckoning of seasons and time.¹⁴ They named and were aware of many of the constellations. In addition, they knew that the circumpolar stars called "indestructible" never set and that certain stars that rose on the eastern horizon were used to signal hours of the night.¹⁵

There is scant evidence which shows the stars being used as a means of finding direction. In the present period, we suggest the ancients must have achieved the precision displayed in the pyramids by use of the stars, for we know their movements and are able to devise clever means that may duplicate the results shown at the first

and second pyramid at Giza. However, the ancient surveyors, lacking our knowledge, might instead have selected the sun. It has an advantage which to them would have been paramount—its shadow. Every object casts a shadow and the constant movement and change of its length during the day must have stimulated their curiosity to the point where a study would have revealed its patterns.¹⁶ In addition, unlike the stellar methods, which require the positions of the instruments used to be accurately transferred to the ground by a plumb line, the shadow is already there. Therefore, it is worthwhile to examine the problem from a fresh point of view, in order to establish a method the Egyptians may have used, bearing in mind that, whatever their means, it was very simple, very obvious, and based on a principle with which they were familiar.

Before a method of utilizing the shadows is described, we should establish that orientation to the meridian was sought after and not an accidental occurrence. This is clearly disclosed in the list of structures in Table 3.¹⁷

A review of Table 3 clearly shows that while some pyramids are oriented with astonishing precision, others are not. The worst example of orientation is displayed by Zoser. However, this was the earliest structure built, and the orientation method may not have been fully realized. If the Zoser pyramid is eliminated from consideration, we have an average deviation of 27' 54" from true north. Therefore, we are not seeking perfection in an orientation method, but one which will account for the errors observed. Assuming the same method was always used, if it always produced precise results, it could not have been the one used by the Egyptian surveyors, unless, as Žaba suggests, these errors are not a reflection of a poorly applied astronomic method, but rather the result of an imperfectly applied geometric procedure. However, considering that Žaba has also found the geometric method to be incredibly accurate,¹⁸ let us assume, for the

¹³ S. Clark and R. Engelbach, *Ancient Egyptian Masonry* (London, 1930), 66.

¹⁴ Parker, op. cit., 706.

¹⁵ Parker, op. cit., 711.

¹⁶ R. W. Sloley, "Primitive Methods of Measuring Time with special reference to Egypt," *Journal of Egyptian Archaeology* XVII (London, 1931), 168.

¹⁷ Žaba, op. cit., 11, 12.

¹⁸ Žaba, op. cit., 61, 62.

TABLE 3 Orientation to the Meridian in Old Kingdom Pyramids

King	Dynasty	Date	Place	Deviation from True North	Base
Zoser	III	2778	Saqqara	3° East	125m W-E 109m S-N
Seneferu	IV	2723	Meidum	24'25" West	144m
Seneferu	IV		Dahshur (N.Pyr.)	9'12" West	188.9m
Cheops	IV		Giza	2'30" West 5'30" East 1'57" South 2'28" North	230m
Chephren	IV		Giza	5'26" West	108m
Mycerinus	IV		Giza	14'3" East	
Sahure	V		Abu Sir	1°45' West	78.35m
Neferirkare	V		Abu Sir	0°30' East	109.6m
Niuserre	V		Abu Sir	0°?*	78.8m
Niuserre	V		Abu Gurab Solar Temple,	-1°	6m

*No deviation is discernible on the large map published in *Wissenschaftliche Veröffentlichungen d. D. Orient-Gesellschaft*, 7.

moment, that the error resulted from an inaccurate astronomic method. In addition, if we can show an astronomic method with an inherent range of error close to that displayed, possibly we will have found the method that the ancient surveyors used.

The astronomic method is suggested by a chart that comes from the Royal Tombs of Thebes and is shown in fig. 1. Here, the sky goddess Nut, with an exaggeratedly long body, is bent over touching the ground, with her head westward and legs to the east. She is shown to ingest the sun, which proceeds to travel through her body, only to be given birth anew 12 hours later. Below her, holding a vertical line that bisects the sun's path is Horus the falcon-headed god.¹⁹ Žaba feels this line, which bisects the distance between east and west, represents the meridian. If so, this is an indication that the Egyptians knew the meridian was the center of the sun's path as it crossed the sky. Although this scene comes from a period later than the Old Kingdom, it is one of the few that record ancient Egyptian thought on the heavens.

¹⁹ Žaba, op. cit., pl. IV.

In view of this, let us re-examine the method rejected by Žaba as being too imprecise, that is, finding direction based on the movement of the sun and the shadow it casts.²⁰ The importance of the sun to primitive man is explained by Hogben when he says:

The groupings of days to make a year was also recognized by the behavior of the sun's shadow. The sun's shadow always pointed in one direction at midday when it was shortest. The noon shadow divided the horizon by a line, going from what we call north to south. At different seasons the sun rose and set far towards the north or south of the horizon, when the noon shadow was respectively shortest and longest. The day of the shortest noon shadow was the summer solstice. A year was also recognized as the number of days between one summer solstice and the next. The days of the vernal and autumnal equinoxes, when the sun rises and sets exactly half-way between the north and south points on the horizon (i.e. due east and west), were the occasions of

²⁰ Žaba, op. cit., 64.

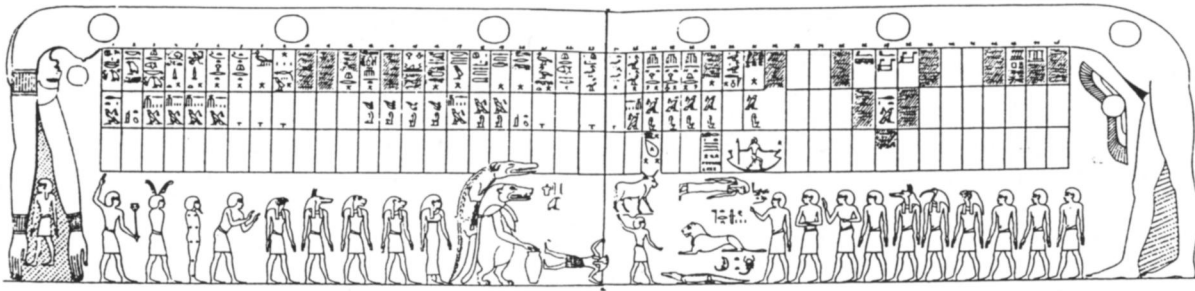


Fig. 1. Chart from a royal tomb at Thebes which shows the sun's path bisected by a meridian line held by Horus.

special rites. Side by side with the observation of the sun's shadow throughout the seasons, Neolithic man was also learning to keep track of his meals and hours of labor by the length of the sun's shadows cast by poles or stone monuments, which he erected to observe it.²¹

Clearly, for early man the sun's path could easily be followed and was a natural time and direction indicator, logically, therefore it was natural to have used it for orientation. Two possible solar methods might be used to find direction—that suggested by Žaba, which consists of finding the shortest shadows cast by the sun, and the other, bisecting the shadows of the rising and setting sun. The theoretical basis of both is that the sun rises and sets in equal and opposite angles to the meridian. As it arcs through the sky, its highest point, and therefore the shortest shadow it casts, is when it crosses the meridian. Therefore, by watching the shadows cast by a gnomon, it is possible to find true north, either by measuring its shadow length, or bisecting the shadows of the rising and setting sun.

Toward this end, Maragioglio suggests a method of finding precise east-west directions by solar means:

A pole was fixed vertically on the levelled rock, for example in the point chosen as the south-east corner of the pyramid. When the sun rose and set in the same day the shadows made on the ground by the pole were traced

and continued on the other side of the pole itself. It is evident how the shadows would have indicated the exact east-west direction only in the equinoctial days, when the evening shadow would have been in line with the morning one. It seems, however, the ancient Egyptians' astronomical knowledge was not advanced enough to enable them to fix the exact date of the equinoxes and the solstices. In any case, the method could have been used in whatever day of the year: in fact, the lines of the shadows and their extensions would have formed angles with the pole as the vertex, the bisection of which would have given a quite precise east-west direction and therefore, in this case, the orientation of the pyramid south side. Lines drawn at right angles would have then easily determined the direction of the other sides.²²

In support of his method, Maragioglio cites an inscription found on the temple at Edfu, which seems to indicate that some of the operations for the tracing of monumental buildings were carried out using shadows projected on the ground and therefore by daylight.²³ Although Maragioglio proposes a simple means to find direction within the capabilities of the ancient surveyors, and cites evidence for its use, the method he describes has inherent problems that may result in false readings. Even if the shadows cast by the rising and setting sun are strong enough to be recorded, the rising sun's horizon

²¹ L. Hogben, *Mathematics for the Million* (New York, 1946), 42-44.

²² V. Maragioglio and C. A. Rinaldi, *L'Architettura Della Piramidi Menfite IV* (Rapallo, 1965), 100-102.

²³ *Ibid.*, 100.

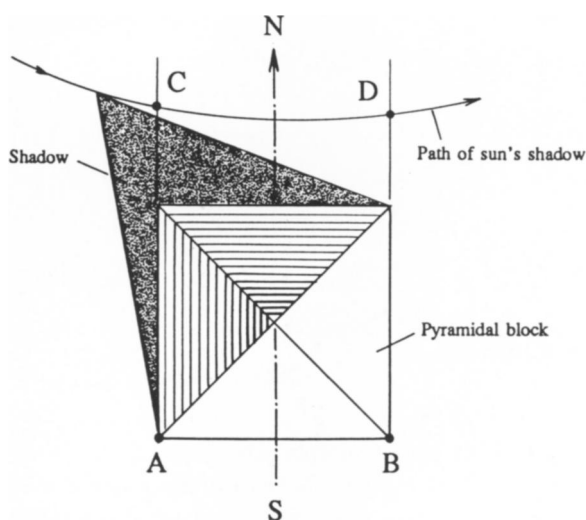


Fig. 2. Neugebauer's method of finding north.

is seldom on an equal plane to that of the setting sun. On land there are depressions, mountain ranges, and other irregularities, which would cause the shadows cast by the rising and setting sun to indicate a false east-west axis. With a slight modification, however, the direction may be more accurately obtained. If means could be found to measure the shadows cast shortly before and after noon when the sun is high in the sky, there would be no uneven horizon to contend with, and the shadows would be clearly delineated because the sun's rays would be more intense.

Perhaps toward this end, Neugebauer, who feels that all stellar orientation theories face serious difficulties says:

There is no bright star exactly at the celestial pole, or . . . rising and setting amplitudes suffer from the poorly defined position of the observer (as well as other practical difficulties). It is therefore perhaps permissible to suggest as a possible method a procedure which combines greatest simplicity with high accuracy, without astronomical theory whatsoever beyond the primitive experience of symmetry of shadows in the course of one day. In short, one can use the shadow of a pyramid as an excellent instrument for orientation. All one has to do is to place an accurately shaped

pyramidal block (e.g. the capstone of the pyramid under construction) on the accurately levelled ground which will eventually carry the monument [fig. 2]. Let its square base be oriented according to a reasonably accurate estimate of the SN/EW directions. Then one observes the path of the shadow cast by the apex of the pyramid from some time before noon to some time after noon. This path describes a curve (which we now know to be a branch of a hyperbola, concave toward North in the winter half of the year, concave toward South in the summer, a straight line at the equinoxes), which will intersect first the western, then the eastern base of the pyramid or a straight continuation in a northerly direction. If these points of intersection are at different distances from, e.g., the south corners (AC and BD respectively), then the orientation is not yet correct. A slight turn of the base and repeated observations on the next day will improve the situation. Not only can this process be repeated many times until high stability is reached but by waiting some weeks one utilizes different tracks and thus in effect averages small errors of individual observations. For example, observations scattered over half a year would lead to a neat set of mid-points between the two parallel base sides providing the desired SN direction. . . .

He goes on to say that any accurately shaped pyramidal model can be used as a gnomon.²⁴

Neugebauer's system will give more accurate results than Maragioglio's, for it does not depend on the moment the rising and setting sun breaks the horizon. In Neugebauer, when the shadow is cast from the apex of the gnomon to the ground, the sun is high in the sky and its rays are not subject to interference from uneven terrain. Although he uses the same theoretical basis as Maragioglio, his method of execution requires a gnomon in the form of a pyramidion with an apex precisely equi-angled and equidistant from the corners of its base. This gnomon is difficult to duplicate and completely unnecessary.

²⁴ O. Neugebauer, "On the Orientation of Pyramids," *Centaurus* 24 (Copenhagen, 1980), 1-3.

The best instrument to observe the sun's shadows is the most ancient astronomical instrument of all, a simple vertical pole. Even our language reflects its relationship to the meridian: the word "pole," in the sense of north pole, is derived from it.²⁵ Indeed, the meridian is a derivative of it; looking south, the observer measured the shadows of the sun at noon, looking north he measured the transit of the circumpolar stars.²⁶ Its value has been universally recognized; it has a long history of use in China,²⁷ it was used in recent times by the American Indians,²⁸ and it is still used by Borneo tribesmen.²⁹

To find direction in India in the 7th century (fig. 3), a pointed vertical pole (gnomon) was centered in a circle drawn on a levelled surface. In the forenoon, when the shadow tip of the gnomon which had been beyond the circle, shortens and reaches the circle, it is marked. In the afternoon of the same day, it is once again marked as it lengthens and leaves the circle. The point at which the shadow entered the circle is westward, and where it left, eastward. The north-south direction can be found by taking the mid-point of these two marks and aligning it with the base of the gnomon.³⁰ This can be done without geometry, i.e., simply by halving a cord, as shown in fig. 4 (this is how the geometric term, "chord of a circle," was derived).

The time of day is indicated by the shadows angled in relation to the meridian, while the solstices or time of year may be found by careful measurements of the shadow's length on the meridian. When using a short gnomon, it is easier to find direction by bisecting the two points of a circle than to try to judge the shortest shadow, for it is difficult to see the almost imperceptible differences in length. Faced with this problem, the Chinese built the Tower

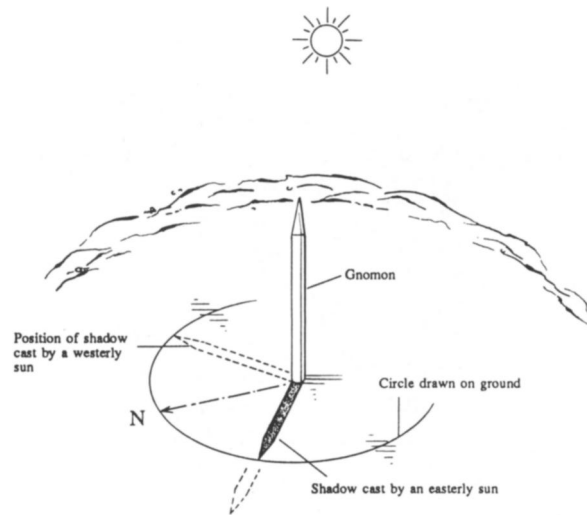


Fig. 3. Indian circle method of finding north.

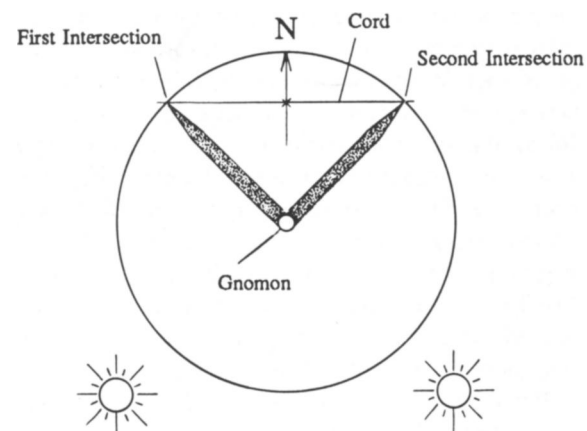


Fig. 4. Method of using a cord to find the center of both intersecting points.

of Chou Kung, upon which they placed at a height of 40 feet (12.19 m), a horizontal bar which acted as a gnomon. A history of the Yuan Dynasty, the *Yuan Shih* gives an analysis of the Chinese gnomon:

When a gnomon is short, the divisions on the scale have to be close together and minute, and most of the smaller divisions below feet and inches are difficult to determine. When a gnomon is long, the graduations are easier to

²⁵ E. C. Krupp, *Echoes of the Ancient Skies* (New York, 1983), 6.

²⁶ J. Needham, *Science and Civilization in Ancient China* (Cambridge, 1959), 231.

²⁷ Needham, op. cit., 284.

²⁸ Krupp, *Echoes*, op. cit., 88.

²⁹ Needham, op. cit., pl. XXX, fig. 111.

³⁰ P. Singh, *Stone Observatories in India*, Research series 11 (Bharata Manisha, 1978), 36; and L. Hogben, *Mathematics*, op. cit., 55.

read, but the inconvenience then is that the shadow is light and ill-defined, making it difficult to get an exact result. In former times, observers sought to ascertain the real point by using sighting tubes, or a pin point gnomon and a wooden ring, all devices for easier reading of the shadow mark on the scale. But now with a 40 foot gnomon, 5 inches (12.7 cm) of the graduation scale corresponds to what was only 1 inch (2.54 cm) previously and the smaller subdivisions are easier to distinguish.

The *Yuan Shih* goes on to describe a shadow definer used in conjunction with this gnomon; a leaf of copper 2 inches (5.08 cm) wide and 4 inches (10.16 cm) long, in the middle of which is a pin-hole. This leaf was mounted in a frame that would permit it to be moved along the ground, on a north-south axis and turned at right angles to the incident shadow. When the pin-hole is first seen to meet the light, it acts as a lens; and an image, no bigger than a grain of rice, is projected on the ground, in the center of which the gnomon can be seen. With this 40 foot gnomon and ingenious shadow definer, the Chinese in the thirteenth century were able to find the summer and winter solstice shadow length with an accuracy of four decimal places (respectively, 12.3695 feet [3.77 m] and 76.7400 feet [23.39 m]).³¹

In India with gnomons 90 feet high (27.43 m) a different method was used; the ill-defined shadow was sharpened by curving the surface upon which the shadow fell, for the closer the incident shadow is to a right angle, the more clearly it is defined. This enabled them to read the shadow time with an accuracy approaching 2 seconds.³²

We can reasonably assume that if the Egyptians made use of the gnomon to indicate time of day, or direction, they would have experienced the same problems that others did; that is, they too may have had need of a device to help define the shadow tip of the gnomon. After reviewing the available possibilities, I have selected and successfully used a device based on an ancient instrument called a *bay*. It consists of

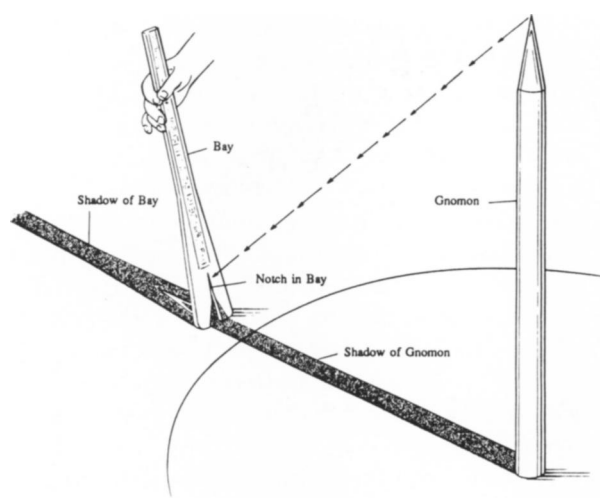


Fig. 5. Manner of defining tip of shadow by use of a bay.

the middle rib of a palm leaf which is slotted at the broader end, and inscribed, "indicator for determining the commencement of a festival and placing all men in their hours."³³

The bay has generally been considered an instrument for sighting the stars; however, judging from the specimen preserved in Berlin,³⁴ Žaba felt it would not be possible to do so with precision, probably because the slot is irregular and asymmetrical. However, if instead of peering through the notch at an object, as is generally supposed, it is placed notch-down and angled to frame the shadow tip as it impinges the surface, it seems to help reduce the fuzziness by blocking the amount of light around the shadow and the reflection off the surface. Neither the shape nor symmetry of the notch is important, as equal success is achieved with a wide range of notches. As shown in fig. 5, the bay is easily positioned at the tip end of the shadow, for it can be clearly seen falling on the surface of the bay facing the gnomon. Drawings of how the shadow tip appears before and after the bay is used are shown in figs. 6 and 7 respectively.

To further understand the problems faced by the ancient surveyors, I have run a trial on

³¹ Needham, op. cit., 296-99.

³² Singh, op. cit., 90, 91.

³³ Soley, op. cit., 169.

³⁴ Personal communication and photograph from Dr. J. S. Karig, Berlin Museum; and Žaba, op. cit., 72.

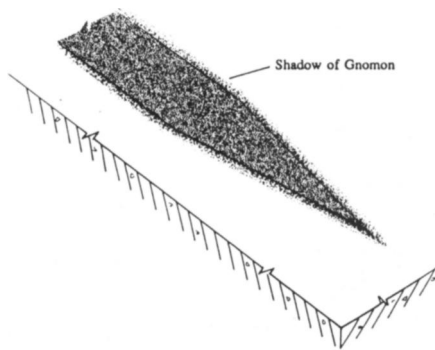


Fig. 6. Undefined shadow tip.

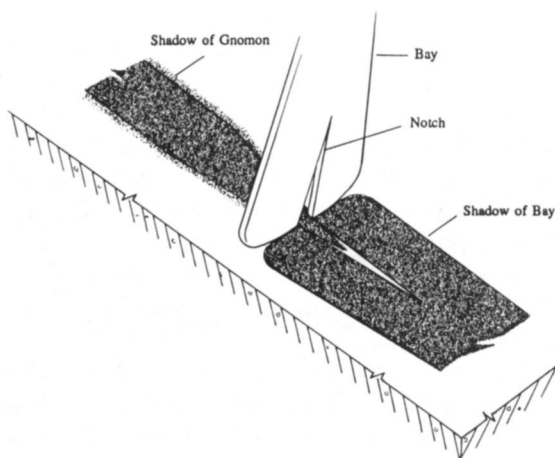


Fig. 7. Defined shadow tip.

finding direction with the Indian circle method, using a pointed gnomon 60 cm high, placed perpendicular to a leveled surface. To make a circle, I rotated a rigid transfer stick, with a marker on its distal end, around the base of the gnomon, striking a 40.3 cm radius. As the tip of the shadow of the rising sun crossed the arc, the bay was positioned to clarify the tip's 2–3 mm fuzzy outline. At the appropriate time, the crossing point was marked, and the procedure repeated as the shadow exited the circle. When the base of the gnomon was aligned with the mid-position of the two points, a reading 19 arc-minutes from true north was recorded.³⁵

³⁵ Wilton, Connecticut, 7 September, 1988, Latitude 41°, Longitude 73°.

With the same gnomon, I attempted to find true north with the method mentioned by Žaba: finding the shortest shadow cast by the sun when on the meridian. This was less successful; for, as described in the *Yuan Shih*, it was difficult to discern the slight differences in shadow length with a short gnomon. However, a longer one would cast shadows whose movements are magnified and therefore easier to read.

In view of this, the ceiling decoration in the tomb of Senmut is interesting. Fig. 8 shows Horus spearing, with what Žaba claims is a line representing the meridian, to the constellation Meskhetiu (represented by a bull). This is tethered to a disc-like object seated on the apex of two diverging lines which extend to a horizontal base. Standing on the base, on both sides of this tall triangle are rows of deities and constellations facing each other. The diverging lines are variously identified as the hoof on the bull's leg, two meridian cords somehow connected to the stretching of the cord ceremony, reins attached to the tail of the bull, or the two extreme positions of a circumpolar star.³⁶ A more likely explanation is that the tall wedge-shaped object represents a gnomon. The disc on its apex, described by Žaba as the star Eta of the Great Bear,³⁷ is probably the sun; for the graphic representations of stars by the Egyptians are distinct and unmistakable and differ from the disc-like form of the sun and moon. A supporting argument is made by Žaba himself when he corrects Borchardt's translation of the text engraved on a *merkhet*, which he claims should read, "I know the movement of the sun disc, the moon disc and the stars, each in its place."³⁸

The tether connecting the tail of the bull to the sun disc seems to symbolically represent the link between the movements of the sun and constellation. An additional point of interest is the figure at the base, left of the gnomon, who seems to be directing the attention of the deity facing him to the summit. The unidentified object held in the outstretched arm of this deity is approximately 30–40 cm long, which is just

³⁶ Žaba, op. cit., 37, 38, 72.

³⁷ Žaba, op. cit., 71, 72.

³⁸ Žaba, op. cit., 57, 58.

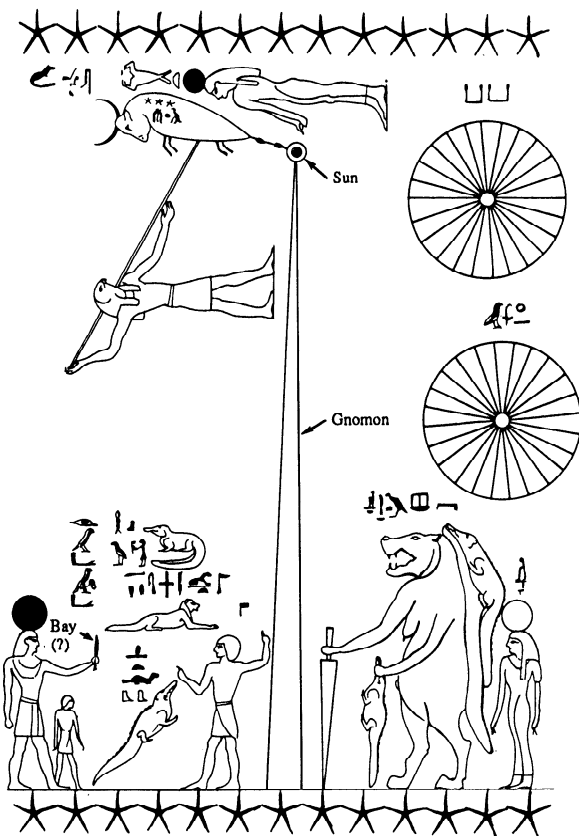


Fig. 8. After the central part of Senmut ceiling's northern panel.

about the length of the bay.³⁹ If it is a bay, the association with the apex of the gnomon and the sun disc is clear. Comparing the height of the figures on the base line, the gnomon's height is about 6–7 m, which is what would be needed to measure shadow lengths with precision. This scene suggests the ancient surveyors might have used the tall gnomon as the Chinese did—to find the solstices by measuring the shortest shadow of the sun as it falls on the meridian.

A similar idea is shown on the Karnak water clock. Here, in the second band, the same wedge-shaped object appears in relation to the constellation, although with less detail.⁴⁰ These scenes show Egypt, along with the rest of the ancient

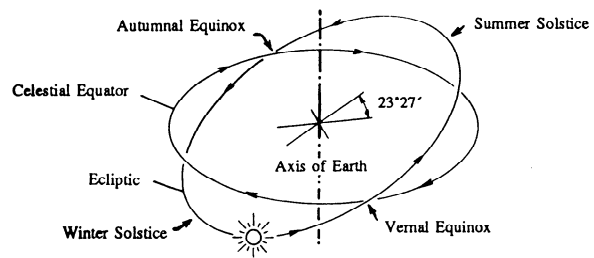


Fig. 9. Angular relation of the ecliptic to the celestial equator.

world, had knowledge and use of the gnomon as an astronomical instrument.

Still, either way of using the shadows to find true north has an inherent problem: the changing declination of the sun during the day may cause the high point of its arc as it travels through the sky to deviate from the north-south axis. As seen in fig. 9, the apparent motion of the sun is not along the celestial equator, which would keep it on an even path around the earth, but along the ecliptic, which is at an angle of $23^{\circ} 27'$ to the equator. This angular relation between ecliptic and equator changes during the year and results in the march of the sun along the horizon between its northern and southern limits (solstices). The angle between them, while greatest at the equinoxes, lessens, and the paths become virtually parallel at the solstices. When viewed on a daily basis during the interval between sunrise and sunset, the angle (ecliptic) of the sun will change slightly in relation to the celestial equator; that is, the sun will rise with one declination and set with another, causing the rising and setting points to be unsymmetrical with the north-south axis. This results in the high point of the arc, being east of the axis about half the year (fig. 10A), and west during the other half (fig. 10B). It coincides with the axis only on the summer or winter solstice (fig. 10C).

The amount of declination depends upon the season and the period of time between sunrise and sunset. The sun's declination appears to change rapidly at the equinoxes (about one degree per day). Therefore, if the interval between sunrise and sunset is about 12 hours, there will be a difference of about 30 arcminutes in declination. It lessens and is significantly

³⁹ F. V. Bassermann-Jorden, *Die Geschichte der Zeitmessung und der Uhren* (Berlin, 1920), pl. 16.

⁴⁰ *Ramses le Grand*, catalogue d'exposition (Paris, 1976), 139–49.

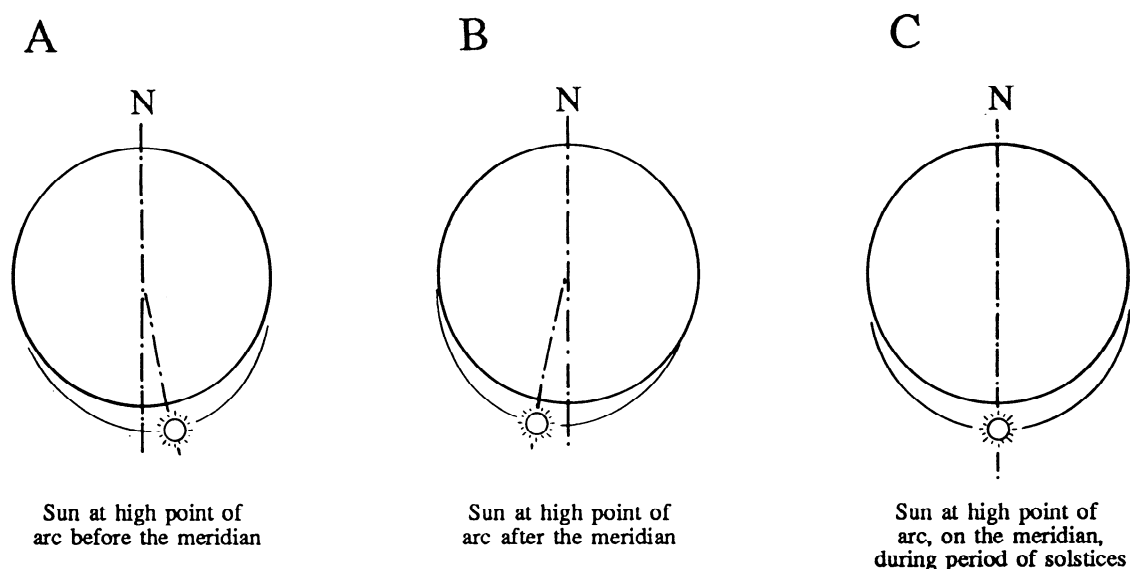


Fig. 10. Relationship of the sun's apparent motion relative to meridian.

reduced at the solstices because at that time the ecliptic is no longer angled to the celestial equator. The time between sunrise and sunset in winter solstice is shorter than in summer and the change in declination then may be as little as 4 arcseconds, while that of summer is 8 arcseconds. To have achieved this kind of accuracy, it would have been necessary to know the solstice to the day. However, if the ancient surveyors worked during the week of the solstices, the error would be only a few minutes of arc.⁴¹ A statue of Harkhebi, an Egyptian astronomer of the third century B.C., contains an inscription which describes not only his familiarity with calendrical reckonings, but his knowledge of the northing and the southing of the sun (summer and winter solstices).⁴² This shows that the Egyptians were as aware of the solstices as were other cultures.

If the surveyors intended to use the solstices for orientation, a good choice would have been the summer solstice, for then the sun's rays are perpendicular to the earth and more intense. Additionally, Sirius, the key calibrator of the Egyptian calendar, after having disappeared

from the sky, reappears in the dawn just before the sun comes up. This heliacal rising fell close to the summer solstice and also signaled the time of the inundation and the New Year.⁴³ With these events occurring approximately at the same time, it may have been a propitious time for Egyptians to start whatever major structure they were planning, particularly the tomb of their god-king.

Just as the Egyptians carefully observed the sky for the heliacal rise of Sirius, the coming solstice may have been signaled by a study of the position of the stars. Indeed, there is a relationship in movement between the sun and the stars, so that all primitive civilizations recognized that different constellations are visible at different seasons of the year. Greeks and Romans, as well as Egyptians and Babylonians, related their agricultural work to the first appearance of conspicuous stars or groups of stars in the region of the sun after they had been invisible for a time.⁴⁴ This may explain the inscriptions at Edfu and Dendera that Žaba found puzzling: two different methods described to determine the north-south orientation, one being quite precise

⁴¹ Personal correspondence from F. C. Krupp, Griffith Observatory, Los Angeles, California, April, 1988.

⁴² Parker, *op. cit.*, 726, 727.

⁴³ Krupp, *Echoes*, *op. cit.*, 21.

⁴⁴ C. Singer, E. J. Holmyard, A. R. Hall, *A History of Technology* I (London, 1954), 796.

(observation of the stars) and the other (observation of the shadow of the sun) less so.⁴⁵ They may have not been different methods to determine the orientation at all. Rather, one method (observation of the stars) may have been used to signal the proper time of year to undertake the orientation process (observation of the shadow of the sun). Although even during this prime period of observation, diffraction caused by haze or wind-blown dust could cause deviations in the sun's rays, which in turn, would result in false readings. These readings, however, may have caused the final result to err as much *towards* as well as away from true north.

Over an extended period of time this orientation method might have been adversely affected by another celestial cycle—precession. This results in a shift of the stars in respect to the earth's axis and is caused by a slight wobble in the earth's axis as it rotates. Its effect is small and is only measurable cumulatively, an entire cycle taking 26,000 years.⁴⁶ The period of pyramid-building lasted only a few hundred years, and precession would have had a minimal effect on orientation.

It would be interesting to know if the times of the solstices were considered when the pyramids were oriented, or whether the sun's shadows were used arbitrarily during the year. This information might be disclosed if a larger number of structures than shown in Table 3 were measured. Without considering diffraction caused by poor atmospheric conditions, orientation by use of the sun's shadows should be close to the average shown by Table 3 (from 0°0'2" to 0°30').

It is obvious the Egyptians intentionally chose to orient the pyramids to the cardinal points, and it may have been done for the following reasons.

Although the stars were a source of wonderment for all ancient people, surely they were secondary in importance to the sun; for their very lives depended on its daily rebirth. In the latitude of Giza, the arc of the sun is about 50 degrees as it travels during the year along the

horizon from winter to summer solstice.⁴⁷ Therefore, if siting a pyramid in relation to the sun was a consideration, it would have been ideal to arrange it to face the mid-point of the sun's travel. By placing the pyramid in this position, the structure would face true north in addition to equinoctial east and west. This would equate the life (east) and death (west) of the pharaoh with that of the sun, and the entrance corridors on the north of the pyramid would be in the direction of the "Imperishable Spirits." These, according to the most ancient belief, held out their arms to the spirit of the dead king to aid him in his ascension to the sky.⁴⁸

The pyramid, whatever its original purpose, may also have served as an immense gnomon. Even in its present state, with the facing stone gone, Mariette, in 1853, used it to determine the time of the vernal equinox within about 29 hours. In those times the inhabitants of the neighboring villages knew that the rays of the setting sun at the equinox grazed the faces of the pyramid; and, Mariette was told, the extremity of the shadow, which was about 3 km long a quarter of an hour before sunset, fell near a granite rock a little north of the village.⁴⁹

With the foregoing, a commonly used method of finding direction has been described and supported by evidence of its use in Egypt, as it was in the rest of the ancient world. While this method was within the technical capabilities of the ancient surveyors, the shadow cast by the gnomon is only a fraction of the length needed to lay out the entire side of a pyramid. The problem presented by a short, accurately found distance, is that it must be projected without loss of accuracy for hundreds of feet, as shown in the first pyramid at Giza. Simply stretching a string over the centers of two closely spaced pegs may lead to error, for the slightest deviation from their center points will be greatly exaggerated as the distance is increased. This is shown in fig. 11 where the stretched cord, although touching both center points, fractionally

⁴⁷ C. G. Ramsey, H. R. Sleeper, *Architectural Graphic Standards*, 6th ed. (New York, 1970), 80.

⁴⁸ Žaba, op. cit., 22.

⁴⁹ G. E. Hale, "The Oriental Ancestry of the Telescope," *Scribner's Magazine* (New York, April, 1925), 398.

⁴⁵ Žaba, op. cit., 65.

⁴⁶ Krupp, *Echoes*, op. cit., 10.

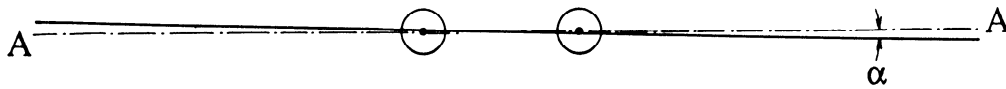


Fig. 11. Problem of accurately extending line over two pegs using a single cord.

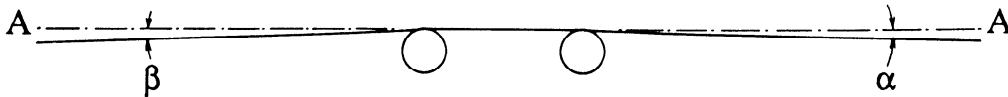


Fig. 12. Problem of accurately extending line tangential to two pegs using single cord.

favors opposing sides, causing the cord to be displaced from its proper position A-A, by angle alpha. The possibility of error also exists when a single cord is placed tangential to both pegs. As shown in fig. 12, the cord, although touching both pegs, is displaced from its true position A-A by angles alpha and beta.

The Egyptians solved this problem with a ceremony called "Stretching of the Cord," which goes back to the 5th Dynasty and probably to Imhotep himself.⁵⁰ Various reliefs of this ceremony show the king representing Thoth and the goddess Seshat,⁵¹ each holding a club in one hand and pole in the other, while facing each other. The poles are joined with a fairly short loop of cord which has been pulled taut by having used them as levers. In addition, the poles appear round, smooth, and of the same diameter, so that both sides of the cord are parallel to each other.⁵²

The pictorial representation shown in fig. 13 is usually accompanied by an epigraph such as that found at Edfu:

I hold the peg. I grasp the handle of the club and grip the measuring-cord with Seshat. I turn my eyes to the movements of the stars. I send forth my glance to Ursa Major . . . stands

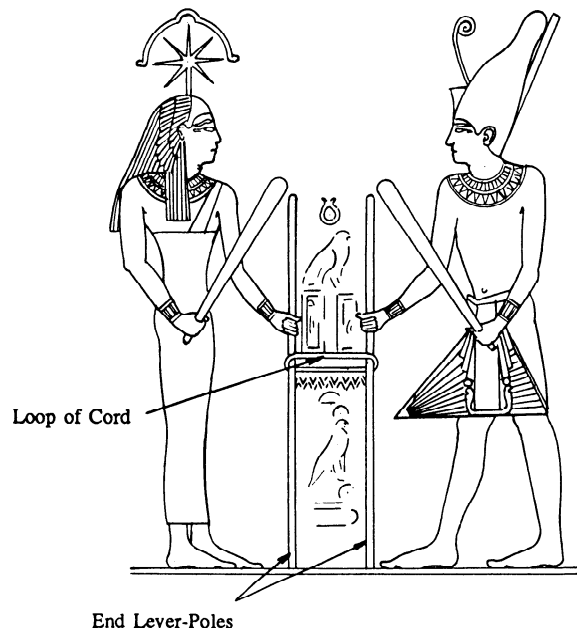


Fig. 13. Typical scene of stretching of the cord ceremony (after Temple of Amada).

beside his merkhet. I make firm the corners of thy temple.⁵³

Or in another epigraph at Edfu:

The king has built the Great Place of Re-Harakhty in conformity with the horizon

⁵⁰ Žaba, op. cit., 60.

⁵¹ Žaba, op. cit., 62.

⁵² Bassermann-Jordan, *Geschichte der Zeitmessung*, op. cit., pl. 17.

⁵³ Sloley, *Primitive Methods*, op. cit., 170.

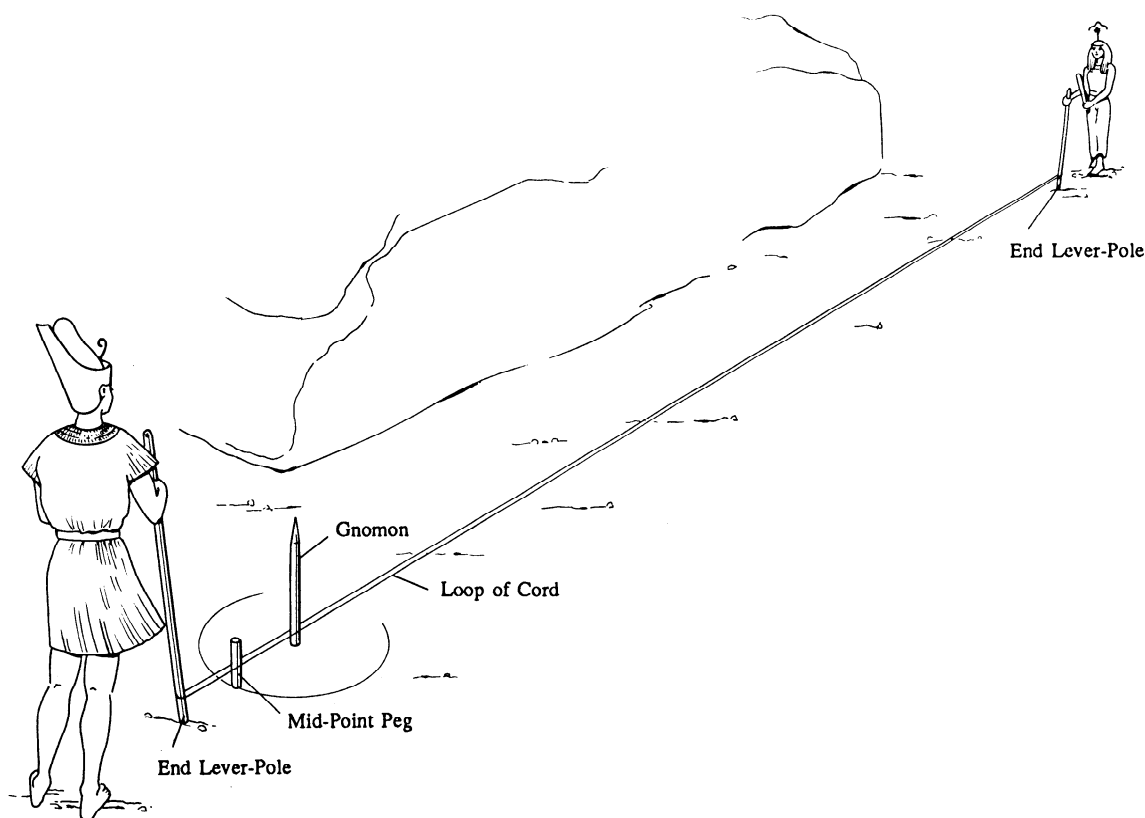


Fig. 14. Actual method of "stretching the cord."

bearing his disk; there the cord was stretched by His Majesty himself, having the stake in his hand with Seshat; he untied his cord with He-who-is-south-of-his-wall, in perfect work for eternity, being established on its angle by the majesty of Khnoum. He-who-makes-existence-run-its-course stood up to see its shadow, it being long in perfect fashion, wide in perfect fashion, high and low in accurate fashion, finished with work of excellent craftsmanship furnished with everything required, sprinkled with gold, decorated with colors; in appearance resembling the horizon of Re."⁵⁴

One of the epigraphs seems to associate the ceremony with the stars and the other associates it with the sun. Ignoring either link and concentrating purely on its technical aspect, let us

see whether there is a practical advantage to stretching a loop of cord.

Žaba feels the two markers and cord, first depicted on the Abu Gurab relief but certainly invented in Egypt much earlier, constituted a primitive compass. In use, he claims, one of the markers was planted in the soil, while the other was used to trace the circumference of a circle. In this way, it was a simple matter to establish the four right angles of the structure with incredibly accurate results. He feels the purpose of representing the two actions of planting a marker as if they occurred simultaneously, and showing the cord too short, was to ensure that both the king and Seshat would fit within the restricted frame of the relief.⁵⁵

I feel the manner of its use was different than that described by Žaba, for I do not believe it was used as a compass. A compass does not

⁵⁴ Žaba, op. cit., 60.

⁵⁵ Žaba, op. cit., 61-62.

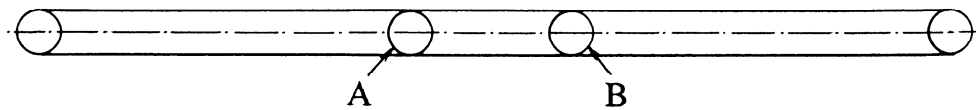


Fig. 15. Method of accurately extending line using pegs and poles of the same diameter with a loop of cord.

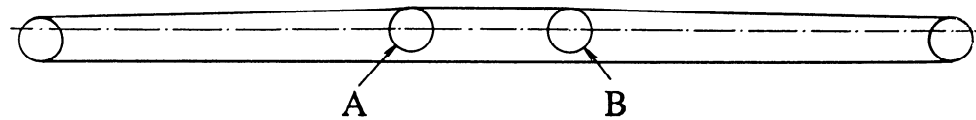


Fig. 16. Condition with both end-poles misaligned on one side.

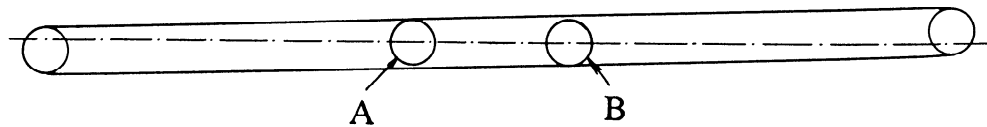


Fig. 17. Condition with end-poles misaligned on alternate sides.

require two lengths of cord to function—one will do very well. The real purpose, I believe, was to extend an accurately found direction with precision. In the pictorial representation shown by fig. 14, where direction was found by the Indian circle method, a peg with the same diameter as the gnomon is placed at the located mid-point. These are then looped with a cord, having its length determined by the proposed structure. At each loop end, a pole of identical diameter as the others is placed. If the gnomon, short peg, and poles are aligned, when the end poles are levered and the cord made taut, they will form the two straight lines shown in fig. 15. The slightest misalignment is clearly displayed by observing the relationship of the parallel cords to the gnomon and peg. This can be rectified by moving one or both of the end poles. In fig. 16, the space on the side of gnomon A and peg B is the result of both end poles being misaligned on the same side. Fig. 17 indicates the end poles misaligned on different sides by showing space on one side of gnomon A and on the other side of peg B.

During my trials of the Indian circle method, two points 40.3 cm apart were established; the south terminus was the base of the gnomon, and the north was the mid-point of where the shadow tip entered and emerged from the circle. By making the loop taut with the two end lever-poles, I have accurately and easily extended the distance to 131 m, although it could not be proved for lack of space, I believe the distance could have been greater without loss of accuracy. For these trials, the closely spaced peg and gnomon were placed near one end of the loop in order to observe the relationship of the cord and adjust it accordingly while holding an end lever-pole. Over this long distance, although the cords will sag, the accuracy will not be compromised; for its horizontal component will be unaffected.

Although the Egyptian pictographs show only the loop of cord and end poles, these are the essential features when representing the "Stretching of the Cord" ceremony; for it was also used when a gnomon was not present. The ceremony was shown in temples not noted for

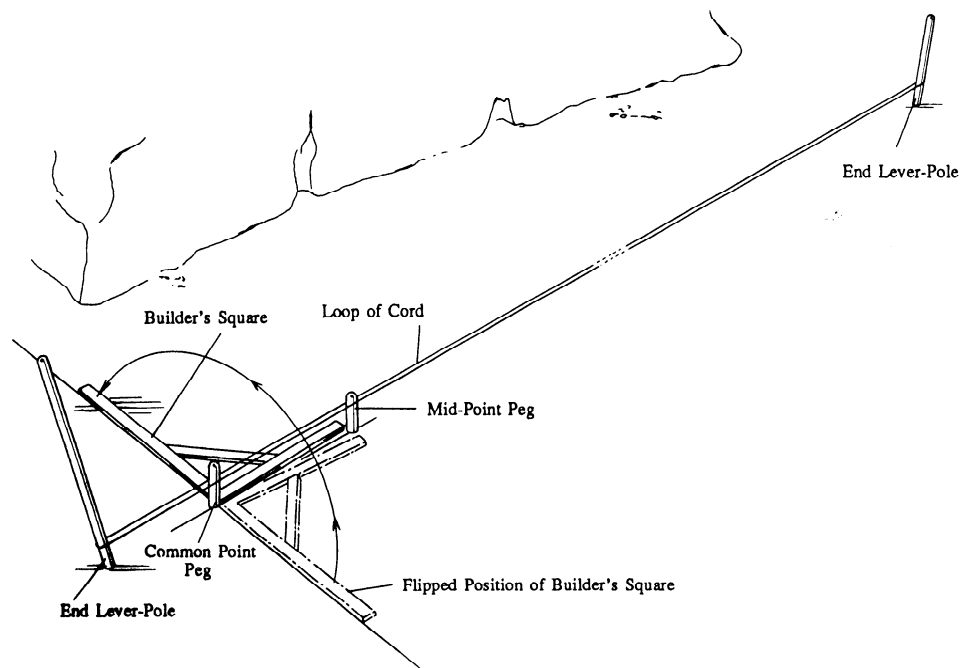


Fig. 18. Method of squaring a corner and extending the line accurately.

careful orientation, which indicates that it may also have been used for only squaring the corners of a structure. As described by Engelbach, and shown in fig. 18, the base leg of a square is placed on a line, and the position of the other leg, which may only approximate a 90° angle, is marked. The square is then flipped over a common point, with its base leg still on the line, but on the other side of the point. The extending leg is once again marked, and the difference, if any, between the two marks is halved. When this bisection is aligned with the common base point, a perfect right angle is produced. The method is shown in practice in fig. 18; however, the angle of its legs is shown excessively acute to more clearly display its theoretical basis. Engelbach claims that in no experiment did the error from the right angle exceed $1\frac{1}{2}$ minutes of arc, which is well within that displayed in the Great Pyramid.⁵⁶ When the square of the corner is found, both the mid-point of the extended legs and the

common base point are pegged. This short, accurately found direction can once again be extended by "stretching the cord." Žaba is correct in concluding the cord stretching ceremony was distinct from the orientation process, but he errs in claiming its function was only to square the corners of a structure.⁵⁷ Clearly, the main function was to accurately extend direction when found by other means. Undoubtedly, the ceremony called "Stretching of the Cord" played a very important part in the foundation ceremony and therefore took on all the trappings of a religious rite.

A method of finding direction based on the natural movement of the sun has been shown, together with a means of accurately extending it. The wonderment is that this astonishing feat of orientation was accomplished with a stick in the ground, a bit of cord, and a shadow.

Wilton, CT

⁵⁶ Clark and Engelbach, *Ancient Egyptian*, op. cit., 67-68, fig. 64.

⁵⁷ Žaba, op. cit., 61-62.