Engineering a Pyramid*

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Introduction

Although building today is accomplished using sophisticated equipment and instruments, the basic methods used have remained the same throughout the centuries. For a structure to attain "true form," precise steps must be taken, whether sophisticated or rudimentary. Thus, an understanding of today's methods can provide insight into the problematic methods used by the ancients in their constructions.

In all trades and occupations methods are used the importance of which often eludes the full understanding of those who are not the practitioners. This could explain why scholars investigating the methods used by the ancient pyramid builders have not heretofore considered methods described in this paper.

Beginning a Construction

The first step in any construction, whether it be an office-skyscraper or a pyramid, is the layout of the building on a selected site in accord with the design and specifications of the architect. This begins with the tedious job of establishing, checking, and rechecking fixed reference points from which control lines can be extended. The best way to store the reference points, which establish control lines in the horizontal planes, is by sinking wooden stakes into the soil so that they will not be disturbed during construction. If confusion arises during any phase of construction, these stakes can be uncovered and used to reestablish control.

Lines connecting these stakes provide the primary horizontal "datum lines" used to control the installation of the building's footings and foundations. In conventional structures, secondary datum lines are further established during construction, with references to the primary, for layout in the horizontal planes of the walls, doors, windows, and other openings and important elements of the construction.

A basic vertical reference is the "sea level benchmark" (the vertical distance of this mark from actual sea level may or may not be known or relevant, depending on the construction). The mark designating this benchmark, used to check height at different phases of construction, could be a nail put into a permanent landmark such as a tree, or a line made on the side of a building unrelated to the construction.

When a building starts to take form, modern builders use a "transit" to transfer these external benchmarks onto the building itself in order to keep the structure level and to control floor and ceiling heights and the heights of all openings. Before the availability of transits, builders used various other means and tools to control level and dimensions. Ancient builders probably used some form of water level or reflected sunlight. In a rectangular hollow building, dimensions could be controlled as the building rose by reproducing or creating control lines at higher levels using relatively simple measurement techniques. However, the means used to control construction of a pyramid is far less obvious and subject to much speculation, simply because of its fundamental geometry.

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Difficulties Faced in Constructing the Pyramids

The primary difficulties in controlling the construction of the pyramids were generated by the enormous size of many of them. The Great Pyramid of Cheops rises to a height greater than a modern forty story building and covers more than thirteen acres.[1] Building such a magnificent structure required solutions to many problems in three main areas.

1. Laying Out the Base

Laying out a level base over such a large area, which was in line with the cardinal directions and in conformity to a "true square," must have been very tedious. In instances where the pyramid was constructed over mounds of uncleared bedrock massif, such as with the Great Pyramid of Cheops, the accuracy achieved is impressive. "A significant consequence of the massif for the layout of the pyramid," suggests Mark Lehner, "was that the square of the base could not be controlled at the outset by measuring the diagonals."[2] Although discussed, this issue will not be explored extensively in this paper.

2. Control of the Form of the Pyramid

Control of the form and dimensions of the acute angular surfaces of a pyramid presents unique problems which are not present during the construction of a hollow rectangular structure. Once a level sub-floor is established for the usual rectangular building, the floor can be used as a benchmark, and the center of the floor can be used as a point from which a control line can be extended, to control the structure through to its completion. In contrast, any such benchmark when used for a pyramid would be destroyed as each successive solid course is added. Yet, using only basic tools such as plumb line, square, level, rule, and string (fig. 1), the builders brought all four sides of the pyramid up to a point directly over the intersecting diagonals of its base while maintaining control of height and keeping all four sides equal, with courses level and corners straight. To achieve this control the stones must have been positioned using fixed references which did not depend on the accumulated accuracies of the cuts of the stones themselves. Even the amazing accuracy of the stones as cut to construct the Great Pyramid[3] could not prevent loss of form if these stones were stacked up into this enormous building without control from external references. The slightest inaccuracy in measurements taken from one of its sides would throw the measurements from the other side off the true line. How the form of a pyramid could have been controlled during its construction is the main topic of the remainder of this paper.

3. Movement of the Stones

Some of the stones used within the internal structure of the pyramids weigh over 15 tons. How these may have been precisely placed is also discussed below.

How Form was Controlled: Some Clues from the Pyramid of Meidum

The pyramid of Meidum was built in three successive phases, with each phase intended as a finished product. The first phase was to construct a seven step pyramid, the sides of which were inclined inward at an angle of 75° and dressed in fine Tura limestone.[4] The second phase was to overlay the seven-step pyramid with another, larger, eight-step pyramid. This could have been done by using the original seven-step pyramid as a template and measuring out from the faces of the first pyramid to position the new casing stones, laid at angles of 75°. First, the base course would have been expanded in this way, and then each successive course. The third phase was to build a true

[1] I. E. S. Edwards, *The Pyramids of Egypt* (N.Y., 1985) 99. Originally published in 1961.

[2] Mark Lehner, "Some Observations on the Layout of the Khufu and Khafre Pyramids," JARCE 20 (1983), 7.

[3] W. M. F. Petrie, *The Pyramids and Temples of Gizeh* (London, 1883), 13.

[4] Edwards, op. cit., 71.

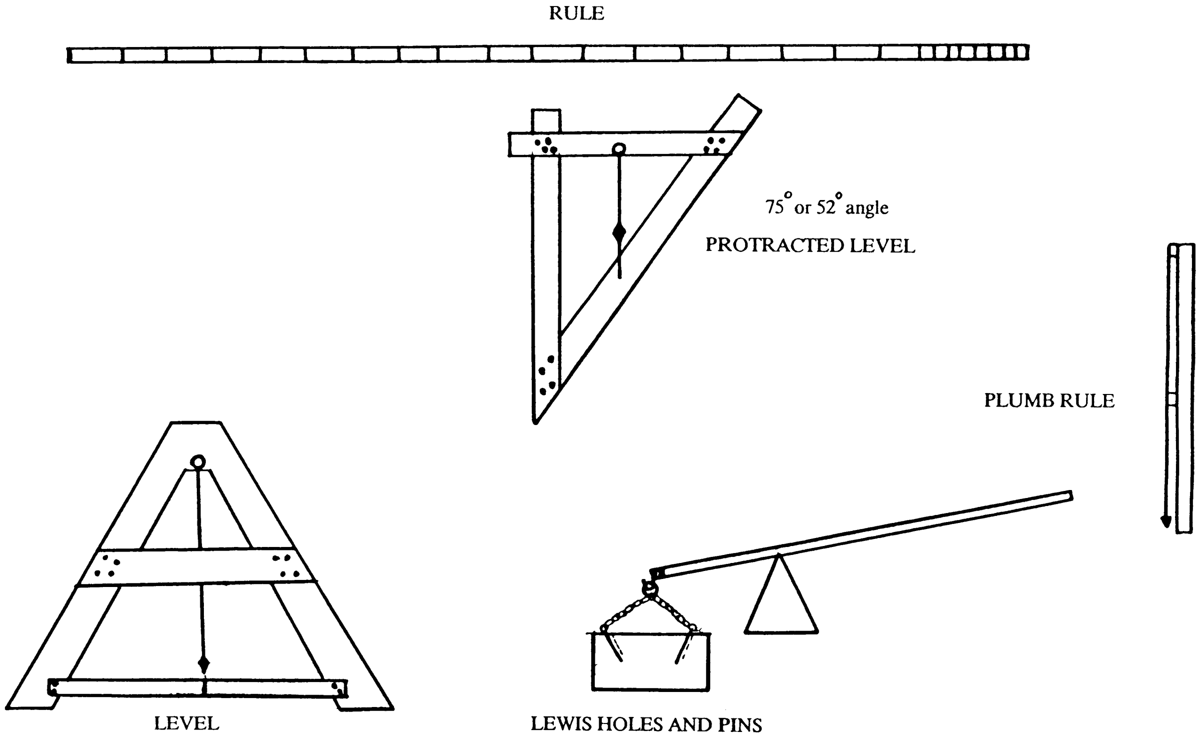


Fig. 1. Basic tools available.

pyramid by filling in the steps with new stones.[5] The outer faces of the casing stones were cut at angles of 52°. This angle could have been determined by pulling a cord down along the edges of the steps (fig. 3). The outer surfaces of these casing stones were set six meters (fig. 2) outward from the edges of the steps of the eight-step pyramid.[6]

These stones were probably laid first, as each course was encased, to control the accuracy of the outer shape and to allow enough room between the steps and the casing stones for workmen to lay the packing stones from behind.[7] This method would also have allowed use of only a single construction ramp for moving and aligning the stones. By using suitable measurements from the step pyramid (fig. 9), the true pyramid could be kept square, without twists and irregularities, so as to form continuous, unbroken, outer surfaces all the way to the apex.

The common method of using the diagonals of the rectangle to control the 90° corner angles could not have been used during the second and third phases because of the presence of the central stepped core. Even in the absence of the central core, the decreasing squares of the ascending courses of any pyramid would require additional reference from some external, fixed control point to avoid misalignment or twisting of the higher, smaller courses in relation to the lower, larger courses. Also, using visual sighting over long distances to control the construction,

[5] Edwards, ibid., 72.

[6] Kurt Mendelssohn, *The Riddle of the Pyramids* (N.Y., 1986), 103. Originally published in 1974. The casing stones could have been set six meters out from the base of the eight-stepped pyramids to allow them to be laid from behind.

[7] Petrie, op. cit., 34. Petrie states that the casing stones were taken to their respective courses already dressed and laid from inside. As Petrie also points out, only one ramp would be necessary and those faces of the pyramid would have been finished as soon as their casing stone had been laid.

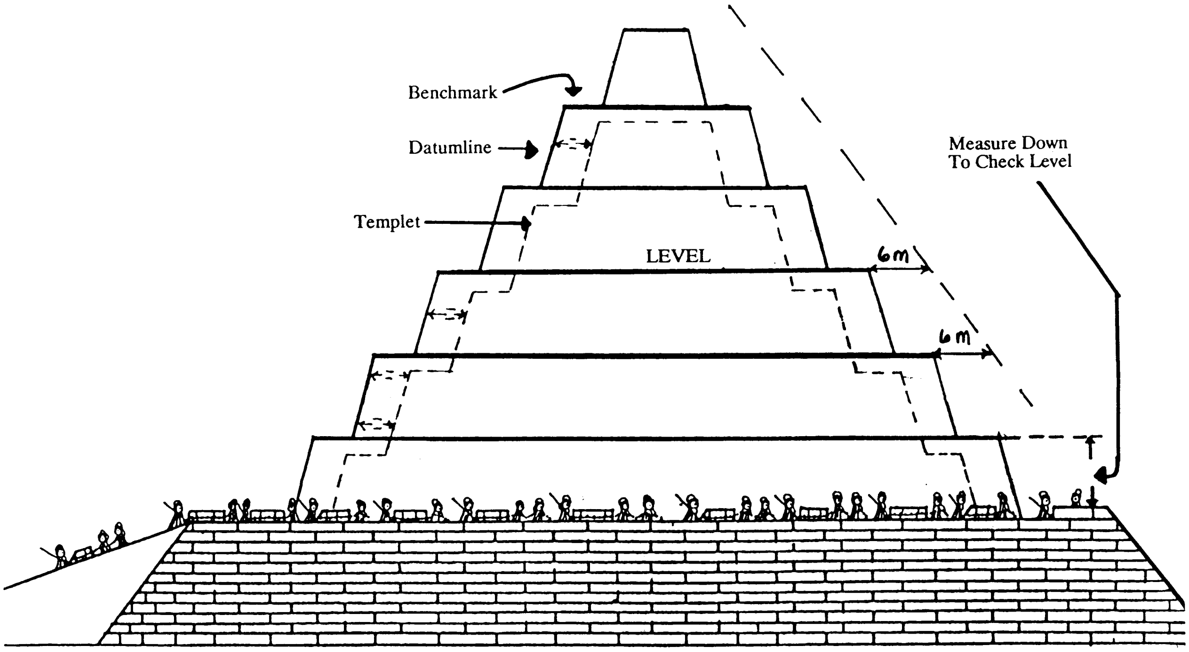


Fig. 2. Shows how stone could have been laid from inside, using simple dimensions from step core to control wall angle and level.

and squaring the structure from reference points which were moved as the structure progressed, would prove too inaccurate.[8]

There seems to be much evidence that the third phase at Meidum was never completed. Some of the casing stones found at the base of the pyramid are dressed, indicating that they were brought to the pyramid and dressed before they were laid.[9]

The presence of a central, stepped, core structure within later "smooth" pyramids suggests that the construction techniques used for the pyramid of Meidum may have established the formula for all future pyramids.[10] Meidum may have been one of the beginnings, culminating in

[8] Martin Isler, "On Pyramid Building," JARCE 22 (1985), 129-42. When referring to control of the pyramidal form, Isler (pp. 129-31) writes about "accurate *measurements* that are required" and "complicated *measurements*" that are needed. "In order to have four sides meet without twists, while maintaining a constant batter, it is necessary to *measure* the structure, as it rises with the utmost accuracy." [Emphasis added] Yet he reiterates Mendelssohn's speculative notions (op. cit. pp. 116-17) on sighting as a guide in building a pyramid, notions which neither consider measurements nor explain how these stone courses were kept level—absolute necessities if the pyramid was to be kept square and thus avoid twists.

Sighting is definitely not, nor has it ever been, a standard or accepted practice in the construction industry. The real source of control on any structure is the measurements and dimensions recorded on the plans and drawings used universally today in all aspects and phases of construction. The ancient Egyptian equivalent to what we do today is the plan referred to by Edwards in *The Pyramids of Egypt*, 254-56.

When a building is under construction in the contemporary world, sightings are taken with a transit, sighting from one point to another to locate and relocate points. *Measurements* are then taken from these points to establish and maintain control.

Measuring is, and always has been, the only means of absolute control on any structure. It is the primary source of control; secondary aides, such as the transit, level, plumb bob, rule, line, and square are used to assist in measuring as needed.

Measurements taken from the wall of the step pyramid to control the angle, dimensions taken down from the steps to control level, are the primary measurements to which Isler should refer. With them and other secondary measuring devices such as a plumb bob, rule, line, and protracted level, the four sides of the pyramid could rise without twists while maintaining a constant batter all the way to the apex.

[9] Mendelssohn, op. cit., 971.

[10] L. Borchardt, *Das Grabdenkmal des Königs Sahure* (Leipzig, 1910), vol. 1, pl. 7. It becomes obvious when

the pyramid of Cheops, of the accumulation of experiences necessary to the maturity of a field of knowledge where basic approaches are first theorized, tested, and then refined.

The Pyramid of Cheops: Progression from Meidum

Although the pyramid of Cheops represented considerable advancement in design and construction from that of Meidum, many similarities attest to the Egyptians' refinement of their concepts of form and construction methods based on past experiences.

Concerning "form," the sides of the pyramid of Cheops slope at an angle of 51° 51′. This has led to much speculation that its form resulted from a plan to make the ratio of the perimeter of its base to its height equal to 2π, which is the same as the ratio of the circumference of a circle to its diameter.[11] The fact that there is no other evidence that the Egyptians of this period had any understanding of the true nature of π suggests that other considerations may have been the true motivation leading to this base to height relationship. This relationship may have been nothing more than a coincidence.

It appears more likely that the external form of later pyramids such as Cheops' resulted from employing structural techniques which had proven successful in earlier pyramids. The angle of the external wall would then have been a consequence of the proven internal structure, rather than the angle being the independent, selected, primary variable which determined the internal structure. Let us develop a rationale on this assumption.

Mendelssohn has observed that, if a pyramid is built using rectangular stones stacked on one another, the directions of the forces generated by the weight of the stones tend to blow out the walls.[12] This is especially a problem if the stones are poorly squared. The Egyptians recognized that these forces could be redirected, giving much improved internal cohesion, if rectangular core stones were laid at an angle sloping inward at 75°. The core stones of Cheops are rectangular but were cut precisely square and laid flat. They were also cut smooth as parallelograms at the location of the buttress walls with internal angles of 75° and 105°,[13] leading to the speculation that such precision provides other functionality in the construction.

Another probable reason for the precise form of these core stones, some of which weigh several tons, has gone unnoticed. The squareness and smoothness of these stones provide surfaces which could have been used as measurement references for accurate positioning of each stone in relation to others, as the courses of an internal, step pyramid are raised and formed. These same surfaces could then have been used to control accurately the positioning of the outer facing stones in the smooth or true pyramid.

The step pyramid encased in a true pyramid obviously determines the shape of the true pyramid superimposed over it. The walls of the pyramid will run parallel to the edge of the steps (fig. 3).

Given the wall angle of 51° 51′ and the internal step riser angle of 75°, we can determine the relationship of the step rise to the step width. Figure 4 shows this ratio to be equal to *2.000, with accuracy to the third decimal.*

It seems highly improbable that this relationship was not planned. Its significance is that, by using this relationship, the stone could be accurately cut and finished to the prescribed dimensions by using rudimentary measuring devices. For example, if the horizontal step width is the primary reference measurement or "master," a secondary measuring device for the 75° face is easily constructed by simply doubling the primary measurement. It is interesting that this rationale leads to the same form for the pyramid that proceeds from the presumption of the use of π as the determinant of the form.

The conjecture that the pyramid of Cheops was built over a precise step pyramid leads to

examining this plate that the buttress walls of the step core stand out as a separate structure from the true pyramid which has crumbled and deteriorated around these accretion walls. One may reasonably draw the inference, therefore, that they were built separately and at different times. Like Meidum, the step core of later pyramids was built first and then converted into a true pyramid in sequence.

[11] Edwards, op. cit., 254.

[12] Mendelssohn, op. cit., 98.

[13] Edwards, op. cit., 260.

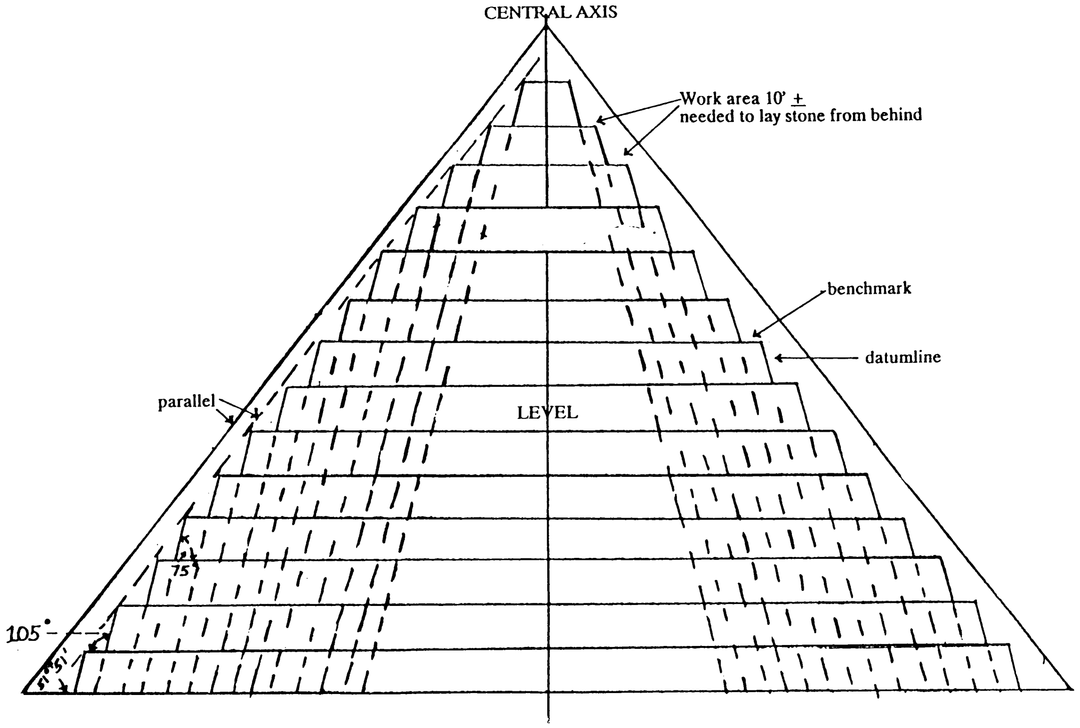
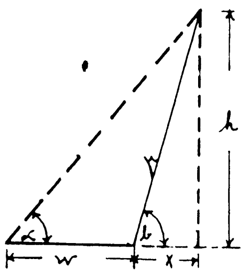


Fig. 3. Pyramid section showing relation of both pyramids.



r = riser face width
w = step face
α = 51° 51'
b = 75°

Determine r/w Solution

$$w = \frac{h - x \tan \alpha}{\tan \alpha}$$

$$h = r \sin b$$

$$x = r \cos b$$

$$w = \frac{r \sin b - r \cos b \tan \alpha}{\tan \alpha}$$

$$\frac{r}{w} = \frac{\tan}{\sin b - \cos b \tan \alpha}$$

for $\alpha = 51^\circ$ 51 and $b = 75^\circ$

$$\frac{r}{w} = 2{,}000$$

Determine $\frac{w + x}{w}$ Solution

$$w + x = \frac{h}{\tan \alpha}$$

$$h = w \frac{(\tan \alpha \tan b\ )}{(\tan b - \tan \alpha)}$$

$$\frac{w + x}{w} = \frac{\tan \alpha \tan b}{\tan \alpha \tan b - \tan}$$

for $= 51^\circ\ 51'$ and $b - 75^\circ$

$$\frac{w + x}{w} = 1{,}518$$

Fig. 4.

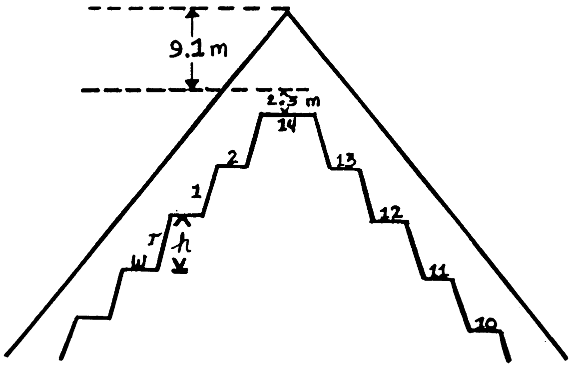


Fig. 5. Approximate location of step pyramid in Cheops.

speculation on the form of the step pyramid itself. Borchardt's measurement of the girdle stone, which he has taken as part of the internal buttress walls, is approximately 5 meters.[14] If we assume that this represents the width of the step (dimension "w" in fig. 4), then, using the "(w+x)/w" relationship of fig. 4 and a half-base dimension of the first course of 115 meters, the inner step pyramid could have fifteen steps. However, the construction of the cap of the smooth pyramid would probably limit the number of steps to fourteen. Using the "h/w" ratio of fig. 4, this would have put the height of the 14th step at 135.2 meters. The original height of the pyramid was about 146.6 meters. Approximately 9.1 meters are missing from the top. Thus, it could be expected that the top of the step pyramid is about 2.3 meters down from the present top of the structure (fig. 5).

Consequent Speculation on the Detailed Construction of the Pyramid of Cheops

The premise that the pyramid of Cheops was constructed using an inner step pyramid to control the form of the outer walls leads to speculation as to how the construction might have actually progressed.

The first construction reference would have been one which established the base of one side, carefully oriented in either the north-south or east-west direction. The accuracy in orientation of the sides to the cardinal directions could only have been done by sighting the rising and setting positions of a star on the eastern and western horizons to establish the first construction reference.[15] Control lines for the other base lines would then have been carefully squared to the first construction references to establish the square perimeter of the base. This would have been done in conjunction with excavating and sculpting the portion of the bedrock core massif that was not removed in clearing the site, to prepare for the eventual placement of the outer casing stones along the perimeter on a level bed. A water level consisting of a flooded trench would have provided control for the depth of any excavation or filling.[16]

Once the base was laid out, the first core stones of the step pyramid would have been placed at a fixed distance from the base perimeter control lines to form the first step. The size of the area between the apothem and the walls of the step would depend on the room needed to lay the packing stones from behind. In the pyramid of Cheops, the large limestone blocks which formed the core were cut at an angle of 75°[17] to form the inward sloping buttress walls rather than square stones laid at that angle (fig. 6). At the lower levels, these core stones would have to have been sculpted to the bedrock core massif that was not removed in clearing the site.[18] The height and level of this first step would have been controlled and checked by measuring up from the base.

As the first step was being formed, ramps, built sideways along the walls of the step, could have provided continuous roadways for moving the stones from ground level to their final positions.[19] A scene found in the tomb of the Twelfth Dynasty monarch Djehutihotep at El-

[14] Mendelssohn, op. cit., 122.

[15] Ahmed, Fakhry, *The Pyramids* (Chicago, 1969), 11. Originally published in 1961.

[16] Edwards, op. cit., 242.

[17] Edwards, ibid., 260.

[18] V. Maragiolio and C. Rinaldi, *L'Architettura IV* (Turin, 1965), 12–13.

[19] Edwards, op. cit., 258.

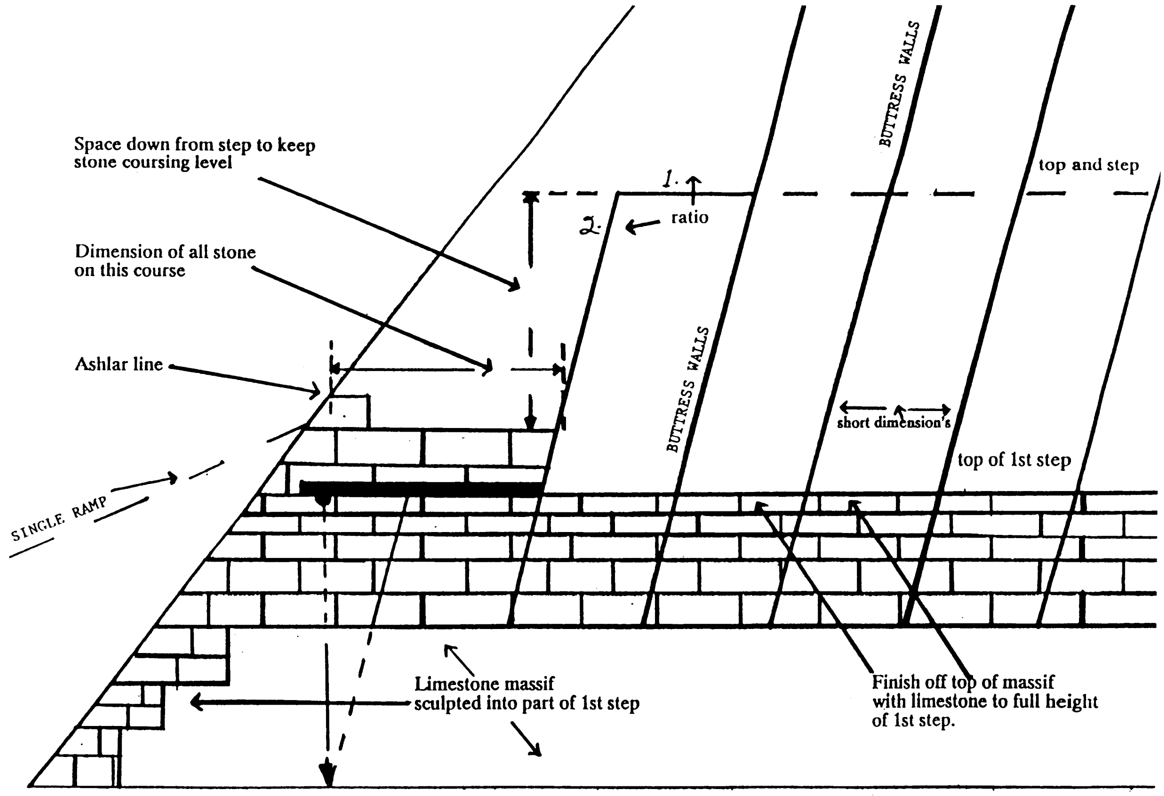


Fig. 6. Pyramid section showing limestone massif sculpted into part of first step.

Bersheh, shows the method of moving his colossal statue.[20] In the illustration the statue is being transported on a wooded sledge. Some men are pulling the sledge while others are pouring a liquid along its path, while still others are carrying long bars which could be used to help pry the sledge loose when it becomes stuck.[21] It seems very likely that this same method would have been used to move the stones of the pyramid over the ramps.

Wet slaked lime, which provides excellent lubricity, was probably spread over the path. (This simple method of moving large limestone blocks has been used even in modern times, and is surprisingly effective.) Water would have then been poured into the slaked lime, as needed, to keep it wet and slick. (It is this phase of the procedure that is so frequently illustrated in tomb scenes of the Old Kingdom and later.) However, if the sledge movement stopped, some assistance would have been needed to get it moving again. Although the wet slaked lime does provide an extremely slippery surface for a mass in motion, it does exhibit a high static or breakaway friction which requires a much higher force to start movement than that required to maintain motion. Thus the need for the men with the long bars.

When the top of the first step was reached, it would have been leveled off, with special attention given to squaring and centering this step with respect to the true pyramid already laid out. The second step would then be started.

[20] P. E. Newberry, *El Bersheh* I, pl. 15.

[21] Fakhry, op. cit., 12. "It is generally thought that this liquid was water, but when examining the copies of the scene, especially those made at the beginning of the nineteenth century, when the colors were fresh, we see that it could be also another material." Fakhry suggests that it was milk. I suggest that mixing the water that was poured on the slaked lime with lime might have been part of the process and given the liquid that appearance of milk.

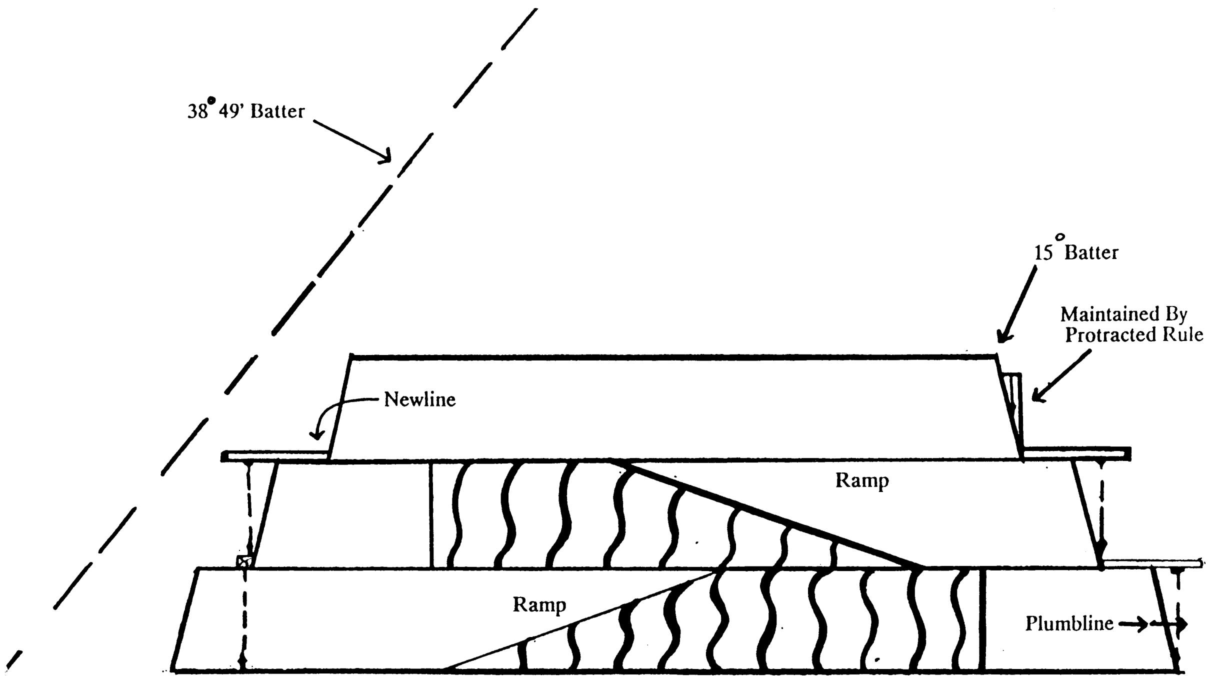


Fig. 7. Simple method of constructing a step pyramid.

Plumb lines would have been dropped from the tops of the outer core stones to control and check the position of their placements in precise relation to the outer core stones of the first step (fig. 7).

The 15° batter (i.e., 90° − 75° of the face of the step with reference to the horizontal) would have made the use of a plumb bob simple and accurate. Then a series of buttress walls, the beginnings of the higher steps, would be laid out measuring back from the existing square outline using only short dimensions. Only the facing blocks of each sloping band of core stones were carefully finished. As this structure gradually rose, an ever-increasing number of buttress walls would be discontinued with increasing height, leading to a succession of steps. The temporary construction ramps, used to move the stones to higher levels would have been built from step to step as soon as it was feasible.

As each step was completed it provided construction references to be used for locating the outer stones of the true pyramid. The locations of the horizontal planes of the steps provided vertical benchmarks, and the walls or risers of the steps provided datum lines for control of key points, as projected on the horizontal plane of the base, as the structure was raised (fig. 8).

Construction of the outer wall of the true pyramid would have been begun by positioning the first block of casing stone to the center of the first course. It would have been laid level and plumb on the line, established earlier, which defined the position of the true pyramid on the site (fig. 9).

The outer face of this casing stone was accurately cut at 51° 51′, the angle obtained by running the apothem of the pyramid parallel with the tops of each step (fig. 3). The measurement from the ashlar line of that stone to the wall of the step pyramid would then have been used by the masons to lay all the stones on that course. Corners would have been kept straight and true by pulling range lines along the ashlar lines of each face of the pyramid. The packing stones would be laid simultaneously behind the casing stones.

The casing closure stone for the course might have been placed from the outside after the

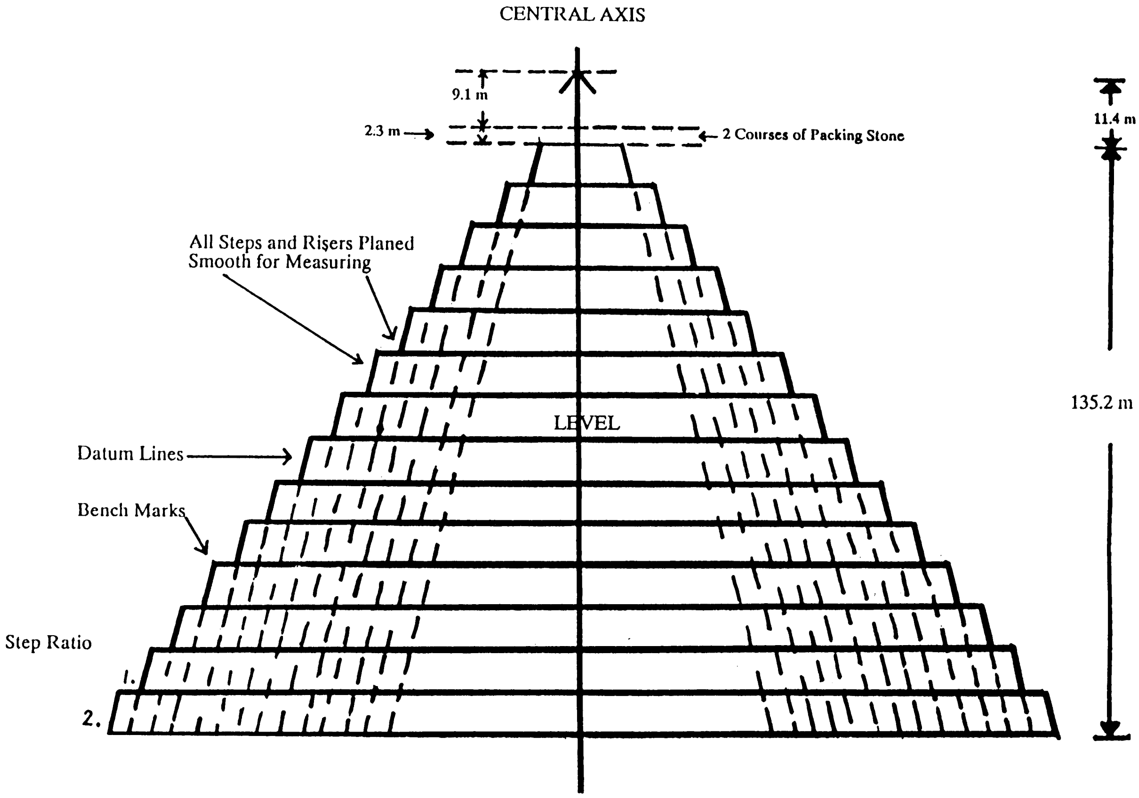


Fig. 8. Shows masonry equal on all four sides of axis; all steps are equal, all risers are equal.

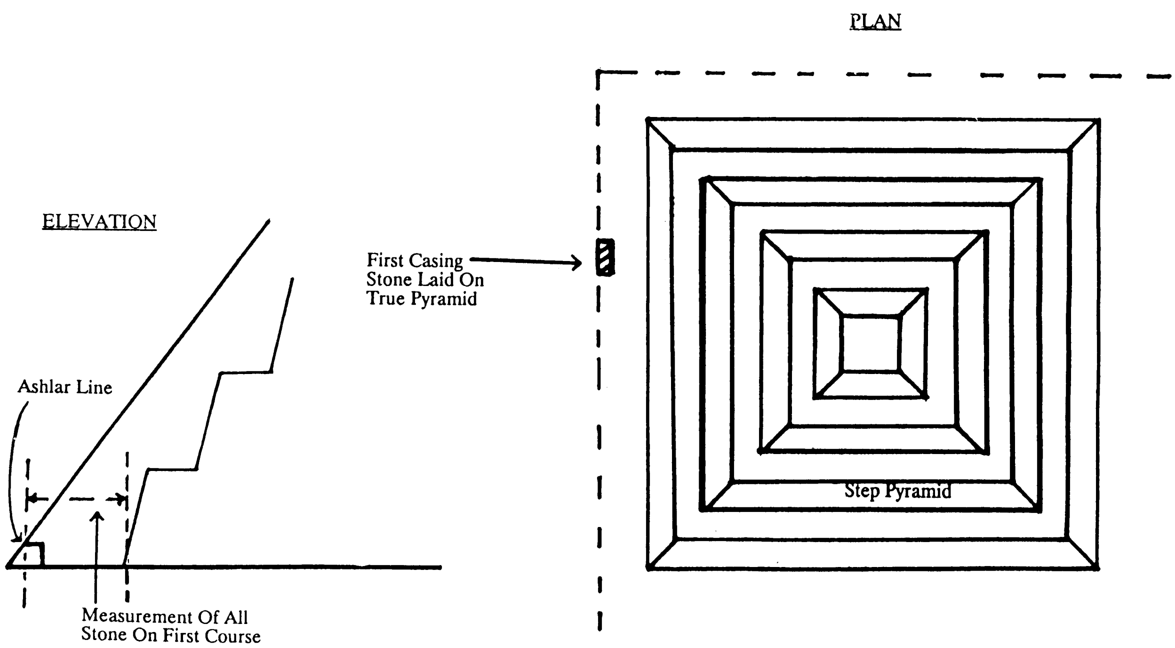


Fig. 9. Showing locations of first casing stone laid on true pyramid.

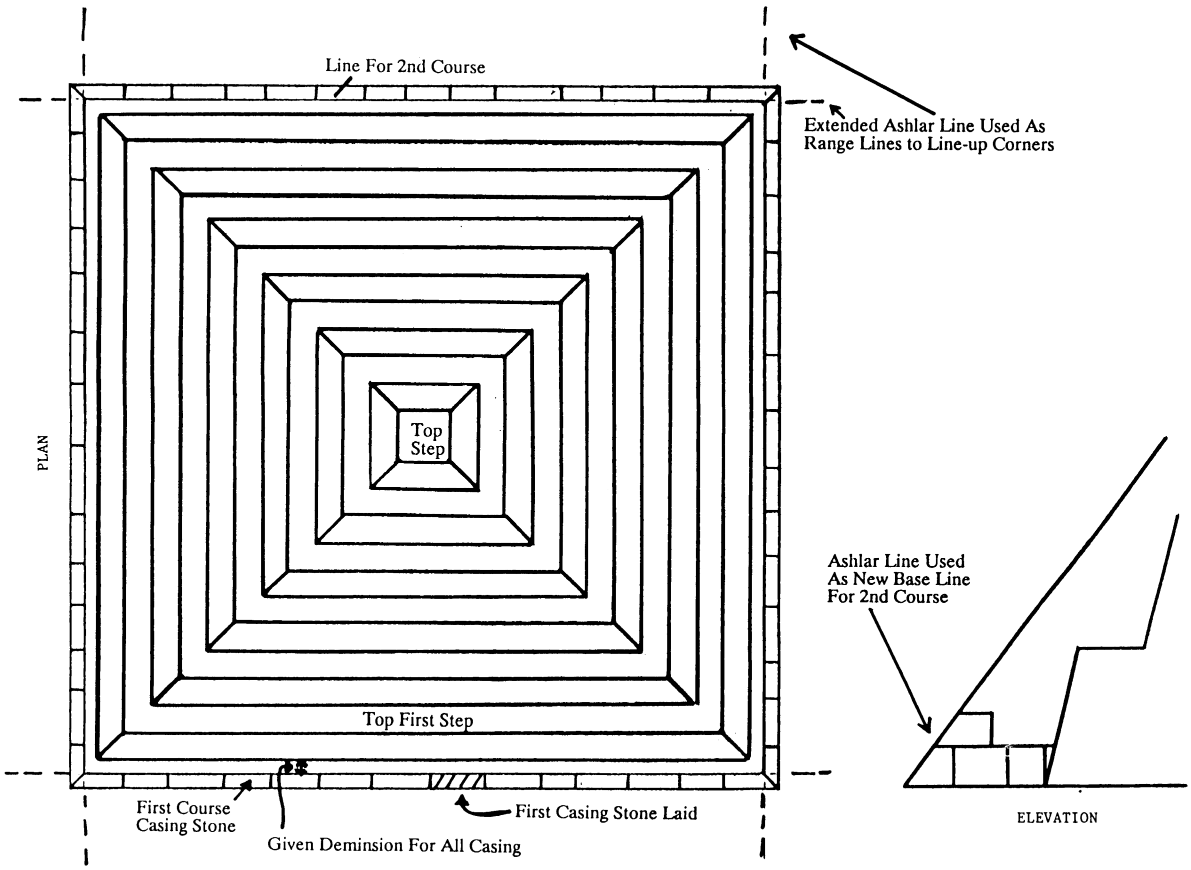


Fig. 10. Shows how ashlar line of first course forms new square base.

packing stone had been laid, or the closure stone could have been laid first and then the few packing stones needed could have been positioned from the next course above, using Lewis holes and pins to lower them into place (fig. 1). When the first course was completed, the ashlar lines of the first course would form a perfect square for use of the base lines for the second course (fig. 10).

A single construction ramp could have been built against one side of the pyramid,[22] then successive courses would have been added with measurements taken down from the top of each next step, with any compensations made which might have been necessary to keep the course under construction level. All packing stone could have been cut in advance, probably using narrower cut stone toward the tops of the steps where the work space between the casing stones and the walls of the steps was necessarily narrower[23] (fig. 11).

By using the ashlar lines of each new course as control lines to regulate the squareness of the structure, and measurements taken down from the steps to control its progressive height, the masons could bring all sides up to a point over the intersecting diagonals of its base, keeping all four sides equal, courses level, corners straight.

Summary

We can never be absolutely certain of the purpose, methods, and procedures of any construction by inference from any analysis of the

[22] S. Clark and R. Engelbach, *Ancient Egyptian Masonry* (London, 1930), 117ff.

[23] Petrie, op. cit., 221.

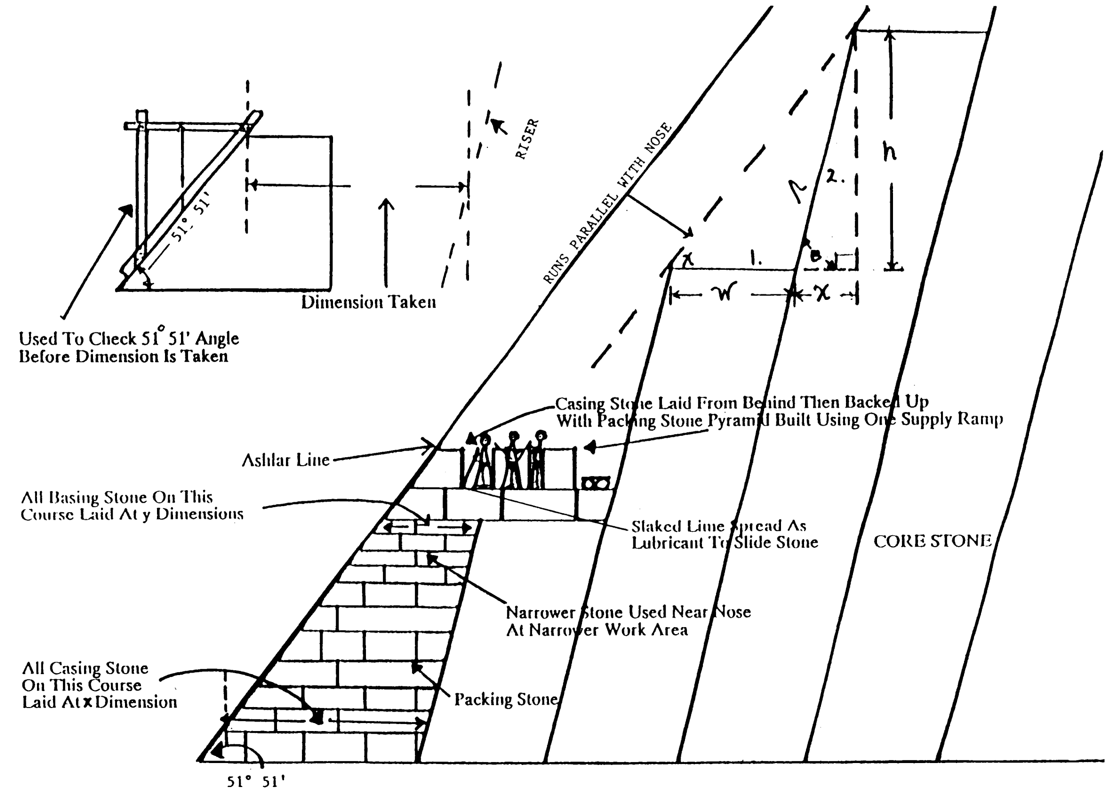


Fig. 11. Pyramid section showing stone being laid from behind.

finished structure itself. This is particularly true for the pyramids since most of the structure is not visible, and only by disassembling the structure can we know for certain even the details on which to base the analysis. However, the requirements, and even the basic methods, used to control any structure were fundamentally the same when the pyramids were built as they are today. By accepting this as a postulate and combining this with a knowledge, from other sources, of the tools available and methods used in other constructions of the same period, the inferences made for the purposes, methods and procedures for the construction of the pyramid of Cheops have a high probability of validity.

Some conclusions reached in this paper are as follows:

1. The pyramid of Cheops demonstrates a culmination of an evolution in the construction of this form. Techniques proven successful in step pyramids were employed and refined in building the pyramid of Cheops, even to the extent of building a step pyramid as the inner construction of the pyramid, to control the form of the outer, smooth surfaces. The conjecture is made that the top of the inner step pyramid can be found at about 2.3 meters down from the present top of the structure.
2. The angle of the external walls, 51° 51′, results from the geometry of the inner step pyramid. It is proposed that this geometry was chosen for considerations of simplicity of measurement and of structural integrity, based on prior building experience. It is further proposed that this results in a base perimeter to height ratio which is only coincidentally equal to 2π.
3. Evidence from other, later sources suggests that slaked lime was used by the Egyptians as an effective lubricant to reduce the force required to move and position large stones. It is highly probable that this method was used during the construction of the pyramid of Cheops as well.

Waltham, Mass.